IRSTI 14.25.07 DOI: 10.59941/2960-0642-2023-2-55-59

Mathematical model of educational event reliability as a random process

Amirova R. A.*[1], Aliyev N. L.[2]
[1]Azerbaijan Technical University, Baku, Azerbaijan,
[2]Azerbaijan State Oil and Industry University, Baku, Azerbaijan

Abstract. This article addresses the application of probability theory in models of education reliability, specifically focusing on the development of events over time. The use of non-monotonic models introduces random variables that exhibit non-monotonic behavior with respect to time, representing the final outcomes of the educational process under the influence of various factors. The article utilizes the Logical-probabilistic method to analyze the reliability of education systems. Mathematical parameters and solutions for educational events are defined, and a model is created to provide a scientific analysis of these events. The article presents a task model involving a system with redundancy and Poisson failure flows. The system consists of a main element and reserves, and their failure and activation processes are described. Furthermore, the article introduces a mathematical model for system reliability using logical algebra and logical reasoning. The reliability of the system is expressed as a logical function dependent on the states of its elements. The article concludes by emphasizing the importance of mathematical modeling and analysis in understanding and ensuring the reliability of educational systems.

Key words: reliability theory, random process, system rejection intensity, operational state, rejection state.

How to cite / Қалай дәйексөз алуға болады / Как цитировать:
Amirova R. A., Aliyev N. L. Mathematical model of educational event reliability as a random process [Text] // Scientific and Pedagogical Journal "Bilim". – Astana: Altynsarin National Academy of Education, 2023. – №2 (105). – P. 55-59.

Introduction

Determining the mathematical parameters of each event and its scientific analysis according to these parameters is very important in the process of understanding it. Mathematical modeling of certain risks, as random processes, and analysis of their solutions are very relevant.

Reliance on the methodological principle "general - special - individual" allows us to distinguish three possible levels of analysis of education from the standpoint of reliability: - at the general level - the reliability of education as a system, - at the level of the special - the reliability of education as a social phenomenon, - at the level of the individual - the reliability of education as a pedagogical process.

The article deals with the problematic issue of applying the theory of probability in models of the reliability of education, reflecting the development of events in time. The problematic of these issues lies in the fact that the final events, the outcomes of the educational process under the possible influence of various factors, as a result of the use of non-monotonic models, are characterized by the implementation of random variables, in the general case with a non-monotonic function of the current time. The meaning of this function characterizes the current probability of the corresponding nonmonotonic event.

Models of the educational process generally reflect real processes - the functioning of elements at different levels of education. The results of the computational implementation of such models are the probabilistic characteristics of the functioning of these tools.

Private processes in such models are implemented using systemic sequences of elementary events that reflect real processes only in the abstract form of the probabilistic characteristics of their occurrence.

Methods

This article uses the Logical-probabilistic method for analyzing the reliability of education systems.

Results and discussion

Let's define the mathematical parameters of the educational event and ways to solve it. Let's create a model to give a scientific analysis of an educational event. Let us define the mathematical parameters of this concept. We characterize events as random events.

Task model. For the mathematical description of a random process and the process of restoring the reliability of the system, the mathematical apparatus of Markov random processes is used. [1,5]

Consider a system with redundancy and Poisson failure flows

$$P_n(t) = \frac{(\lambda t)^n}{n!} e^{-\lambda t}, \quad n = 0,1,2,\dots$$

where is the average number of events over a certain time interval.

1. Let the system consist of the main element A_1 and two reserve A_2,A_3. If element A_2 fails, A_2 is switched on, if A_2-A_3 fails. Before switching on, each of the reserve elements is in reserve and cannot fail. Flow intensity of the main element λ_1; λ_2=λ_3; . All failure flows are Poisson. It is required to determine the reliability of the system. Element A_1 is the current state of the system, that is, a conflict event. A_2 - peaceful conflict resolution, A_3 - conflict resolution through negotiation. The probability of element A_1 is equal to p_1, the probabilities of A_2 and A_3 are equal to p_2 and p_3., respectively.

Let's imagine the process running in the system as a Markov random process with continuous time and discrete states: S_i – element A_i works, i=1,2,3, S_4 – no element works. The system of Kolmogorov equations for such states has the form

$$\begin{cases} \dfrac{dP_1}{dt} = -\lambda_1 P_1, \dfrac{dP_2}{dt} = -\lambda_2 P_2 + \lambda_1 P_1 \\ \dfrac{dP_3}{dt} = -\lambda_2 P_3 + \lambda_2 P_2, \dfrac{dP_4}{dt} = -\lambda_2 P_3 \\ P_1(0) = 1, P_2(0) = 0, P_3(0) = 0 \\ P_1 + P_2 + P_3 + P_4 = 1 \end{cases}$$

By integrating the equations with the initial conditions, we obtain

$P_1 = e^{-\lambda_1 t}$, $P_2 = \frac{\lambda_1}{\lambda_2-\lambda_1} e^{-\lambda_1 t} - \frac{\lambda_1}{\lambda_2-\lambda_1} e^{-\lambda_2 t}$,

$$P_3 = \frac{\lambda_1\lambda_2}{(\lambda_2-\lambda_1)^2} e^{-\lambda_1 t} - \frac{\lambda_1\lambda_2}{(\lambda_2-\lambda_1)^2} e^{-\lambda_2 t} - \frac{\lambda_1\lambda_2 t}{\lambda_2-\lambda_1} e^{-\lambda_2 t}$$

$$P_4 = 1 - P_1 - P_2 - P_3$$

Then the reliability of the system is equal to the sum of the corresponding probabilities:

$$P(t)= P_1+P_2+P_3$$

where *P(t)* - is the reliability of the system, i.e., full decision event.

2. Let the system consist of the main element A_1 and three reserve A_2, A_3, A_4; in case of failure of the main element A_1, reserves are included in the work, A_2 - solution at the next level, A_3 - solution of the problem at a high level, A_4 - Audit is a key factor in solving the problem of reliability.

Flow intensity of the main element λ_1; $\lambda_2=\lambda_3$.

All failure flows are Poisson. It is required to determine the reliability of the system. The probability of element A_1 is equal to p_1, the probabilities of A_2 and A_3 are equal to p_2 and p_3., respectively $p_4 = 1 - p_1 - p_2 - p_3$. Consider the states S_{ij}, where *i* = 1 - the main element is operational, *i* = 0 - the main element is not operational, *j*=1 out of 3 reserve 1 is operational, *j*=2 failed: 2 out of 3 reserve 2 are operational, 1 failed; *j*=3 out of 3 reserve 3 are operational. Probability data $p_{13}, p_{12}, p_{11}, p_{10}, p_{03}, p_{02}, p_{01}, p_{00}$ correspond to states $S_{13}, S_{12}, S_{11}, S_{10}, S_{03}, S_{02}, S_{01}, S_{00}$.

$$\begin{cases} \frac{dP_{13}}{dt} = -(3\lambda_2+\lambda_1)P_{13}, \frac{dP_{12}}{dt} = -(2\lambda_2+\lambda_1)P_{12} + 3\lambda_2 P_{13}, \\ \frac{dP_{11}}{dt} = -(\lambda_2+\lambda_1)P_{11} + 2\lambda_2 P_{12}, \frac{dP_{10}}{dt} = -\lambda_1 P_{10} + \lambda_2 P_{11}, \\ \frac{dP_{03}}{dt} = -3\lambda_2 P_{03} + \lambda_1 P_{10}, \frac{dP_{02}}{dt} = -2\lambda_2 P_{02} + \lambda_1 P_{12} + 3\lambda_2 P_{03}, \\ \frac{dP_{01}}{dt} = -\lambda_2 P_{12} + \lambda_1 P_{11} + 3\lambda_2 P_{02}, \frac{dP_{00}}{dt} = -\lambda_1 P_{10} + \lambda_2 P_{01}, \\ P_{13} + P_{12} + P_{11} + P_{10} + P_{03} + P_{02} + P_{01} + P_{00} = 1 \\ P_{13}(0) = 1, P_{12}(0) = \cdots = P_{00}(0) = 0 \end{cases}$$

The integration of the system can be carried out in the following order: from the first equation we find

$$P_{13} = e^{(3\lambda_2+\lambda_1)t}$$

Then we substitute this expression into the second equation and find P_{12} we substitute P_{12} into the third equation, and so on. Then the reliability of the system is equal to the sum of the corresponding probabilities:

$$P(t)=P_{13}+P_{12}+P_{11}+P_{10}+P_{03}+P_{02}+P_{01}$$

Consider a mathematical model of system reliability in logical algebra with logical reasoning. *P(t)* - the reliability of the system corresponds to the state $S_{ij}=S_{13}$ (where *i*=1 means that the main element is working, *j* = 3 means that all 3 out of 3 backup ones are working).

Let there be some system consisting of the main element and 3 reserve elements. The system starts up with a functional main element that performs the function assigned to the system. The state of the main element is A_1, where A_1=1 if the element is operational, and A_1=0 if it is not operational.

Similarly, we denote the state of the i-th reserve element A_i, $i=\overline{(2,4)}$, where A_i=1 if it is operational, and A_i=0 if it is not operational.

The state of the entire system *P(t)*, where *P(t)=1*, if the system performs the function assigned to it, i.e. is operational, and $P(t)=0$ if it does not perform (inoperable).

The system is operational if only the main element or at least one of the 3 backup elements is operational in it.

System state function

$$P(t)=f(A_1,A_2,A_3,A_4),$$

expressing the dependence of the state of the system $P(t)$ on the states of its elements A_1,A_2,A_3,A_4 at the same arbitrary point in time, can be written as the following logical function, in which the sign V means the operation of a logical Boolean disjunction

$$P(t) = A_1 V A_2 V A_3 V A_4 = 0111111111111111\ (*)$$

This means that an existing failure can now be restored. Through these methods, ways of analyzing and ensuring the reliability of the situation identified by the Markov process are analytically presented.

Conclusion

The article draws an analogy with classical research methods. Based on this, a mathematical model of this event was created and methods for restoring this state as a system were studied. Modeling this random process and restoring its performance provide a scientific approach to solving this process.

The final events, the outcomes of the educational process, as a result of the application of non-monotone models, are characterized by the implementation of random variables, in the general case with a non-monotone function of the current time of the development of the education system. The meaning of this function characterizes the current probability of the corresponding nonmonotonic event.

References

1. **Amırova R. A., Alıev T. N.** Calculatıon of relıabılıty of restored power supply systems // 9ROXPH. – 2022. – P. 108.
2. **Amirova, R. A.** Reliability problem in some algebraic structures // Control and optimization with industrial applications. – 2020. – P. 119.
3. **Amirova, R. A.** Reliability of the dynamics of functioning systems of objects with the structure (2020) Asia Life Sciences, Supp22 (2), pp. 257 - 272.
4. **Ryabinin, I. A.** Logical probabilistic calculus: An apparatus for studies in reliability and safety of structurally complex systems (2003) Avtomatika i Telemekhanika, (7), pp. 178 – 186.

Математическая модель надежности образовательного события как случайного процесса

Амирова Р. А.*[1], Алиев Н. Л.[2]
[1]Азербайджанский технический университет, Баку, Азербайджан
[2]Азербайджанский государственный университет нефти и промышленности, Баку, Азербайджан

Аннотация. В данной статье рассматривается применение теории вероятности в моделях надежности образования, в частности, уделяется внимание развитию событий во времени. Использование немонотонных моделей вводит случайные величины, которые демонстрируют немонотонное поведение относительно времени, представляя конечные результаты образовательного процесса под влиянием различных факторов. В статье используется логико-вероятностный метод для анализа надежности образовательных систем. Определяются математические параметры и решения для образовательных событий, создается модель для научного анализа этих событий. В статье представлена модель задачи, включающая систему с резервированием и пуассоновскими потоками отказов. Система состоит из основного элемента и резервов, описаны процессы их отказа и активации. Кроме того, в статье представлена математическая модель надежности системы с использованием логической алгебры и логических рассуждений. Надежность системы выражается в виде логической функции, зависящей от состояний ее элементов. В заключении статьи подчеркивается важность математического моделирования и анализа для понимания и обеспечения надежности образовательных систем.

Ключевые слова: теория надежности, случайный процесс, интенсивность отказов системы, рабочее состояние, состояние отказа.

Кездейсоқ процесс ретінде білім беру оқиғасының сенімділігінің математикалық моделі

Р. А. Амирова*[1], Н. Л. Алиев[2]
[1]Әзірбайжан техникалық университеті, Баку, Әзірбайжан
[2]Әзірбайжан мемлекеттік мұнай және өнеркәсіп университеті, Баку, Әзірбайжан

Аңдатпа. Бұл мақалада білім берудің сенімділік модельдерінде ықтималдық теориясын қолдану қарастырылады, атап айтқанда, уақыт өте келе оқиғалардың дамуына назар аударылады. Монотонды емес модельдерді қолдану әр түрлі факторлардың әсерінен білім беру процесінің соңғы нәтижелерін білдіретін уақытқа қатысты монотонды емес мінез-құлықты көрсететін кездейсоқ шамаларды енгізеді. Мақалада білім беру жүйелерінің сенімділігін талдау үшін логикалық-ықтималдық әдісі қолданылады. Білім беру оқиғаларының математикалық параметрлері мен шешімдері анықталады, осы оқиғаларды ғылыми талдауға модель жасалады. Мақалада брондау жүйесі мен пуассондық сәтсіздіктер ағындары бар тапсырма моделі берілген. Жүйе негізгі элемент пен резервтерден тұрады, олардың істен шығу және белсендіру процестері сипатталған. Сонымен қатар, мақалада логикалық алгебра мен логикалық пайымдауды қолдана отырып, жүйенің сенімділігінің математикалық моделі келтірілген. Жүйенің сенімділігі оның элементтерінің күйлеріне байланысты логикалық функция түрінде көрінеді. Мақаланың қорытындысы білім беру жүйелерін түсіну және сенімділікті қамтамасыз ету үшін математикалық модельдеу мен талдаудың маңыздылығын көрсетеді.

Түйінді сөздер: сенімділік теориясы, кездейсоқ процесс, жүйенің істен шығу қарқындылығы, жұмыс күйі, істен шығу күйі.

Material received on 10.05.2023